STABLE32 FREQUENCY DOMAIN FUNCTIONS

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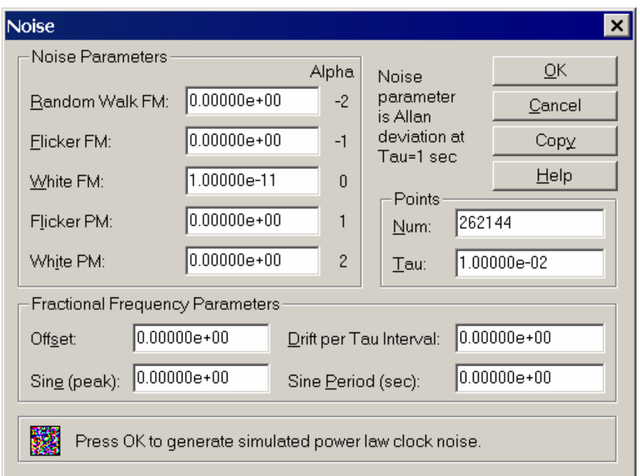
ABSTRACT

This document shows an example of a time and frequency domain stability analysis using Stable32. First, a set of simulated power law noise data is generated using the Noise function. Then the time domain properties of this noise are analyzed using the overlapping Allan deviation with the Run function. Next the same data are analyzed in the frequency domain with the £(f) Power function. Then the expected results are found with the Domain function, and compared against those obtained. Finally, the other power spectral density types are examined, and the various Power function options are discussed.

NOISE GENERATION



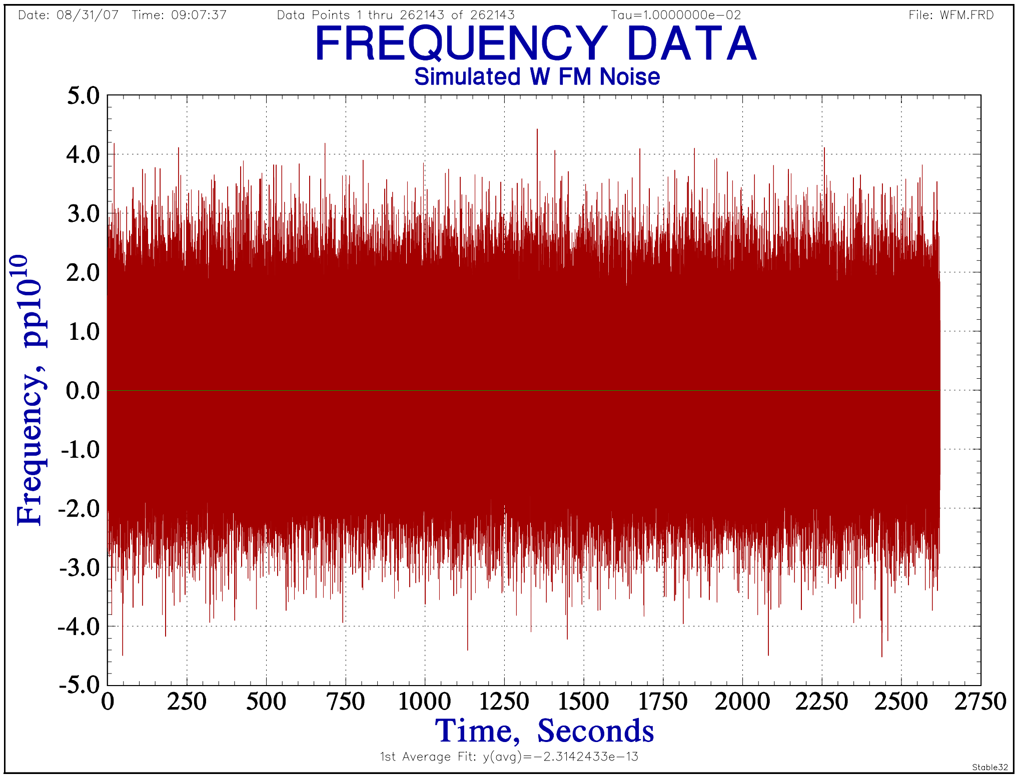
Choose the Noise function, and enter the desired noise parameters into the dialog box controls. For this example, we will generate 2^{18} = 262,144 points of simulated white FM noise with a 1-second Allan deviation value $\sigma_y(1) = 1\mathrm{x}10^{-11}$ and a sampling interval τ = 10 msec. Pressing OK will generate and display a set of phase and frequency noise data having those parameters (each set will be different). The number of points is chosen as an even power-of-2 for the phase data.

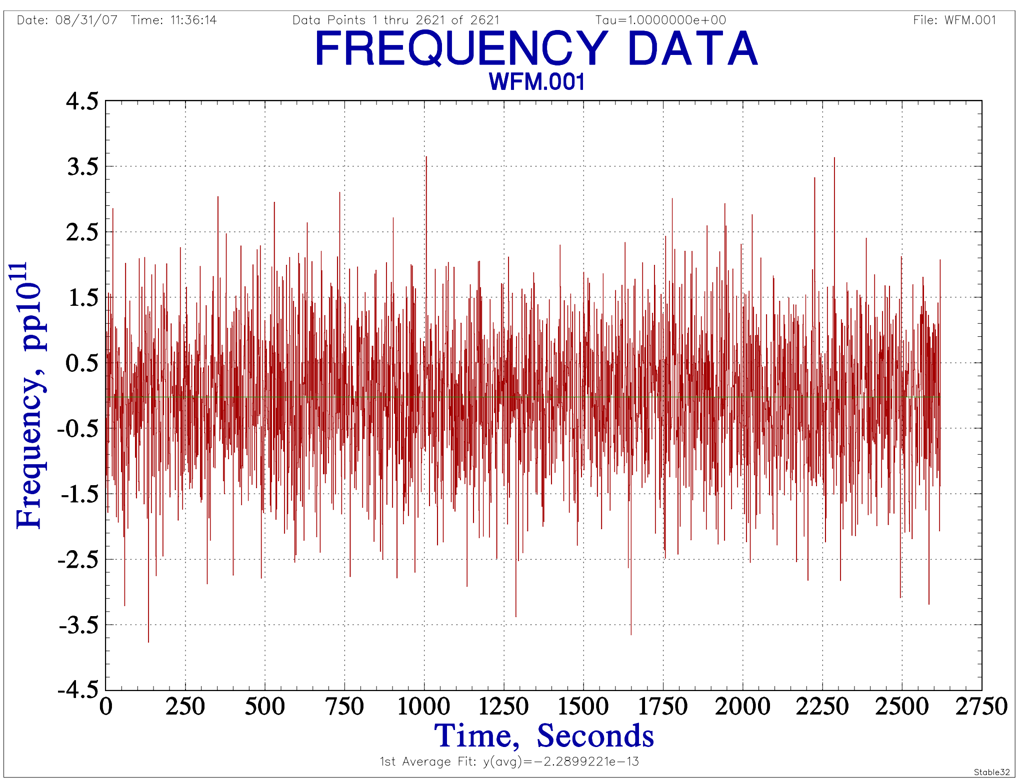
```
Power-Law Noise Generation:

     Points:
          Num=262144
          Tau=1.00000e-02
     Noise Parameters:
          RW FM=0.00000e+00
          F FM=0.00000e+00
          W FM=1.00000e-11
          F PM=0.00000e+00
          W PM=0.00000e+00
     Fractional Freq Parameters:
          Offset=0.00000e+00
          Drift/Tau=0.00000e+00
          Sine=0.00000e+00
          Period=0.00000e+00
```

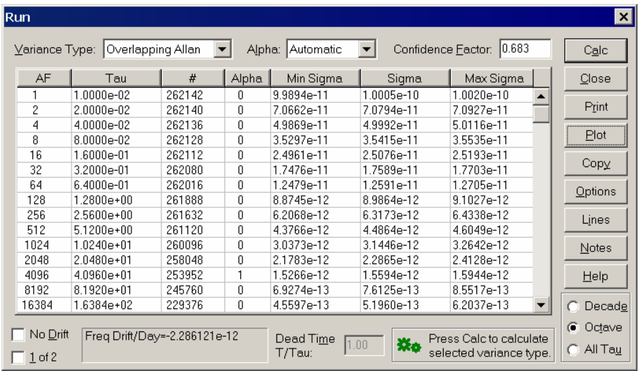
DATA PLOTS

Use the Plot function to display the generated set of simulated white FM noise as frequency data. The lower plot shows the data after averaging by a factor of 100 to a tau of 1 second.



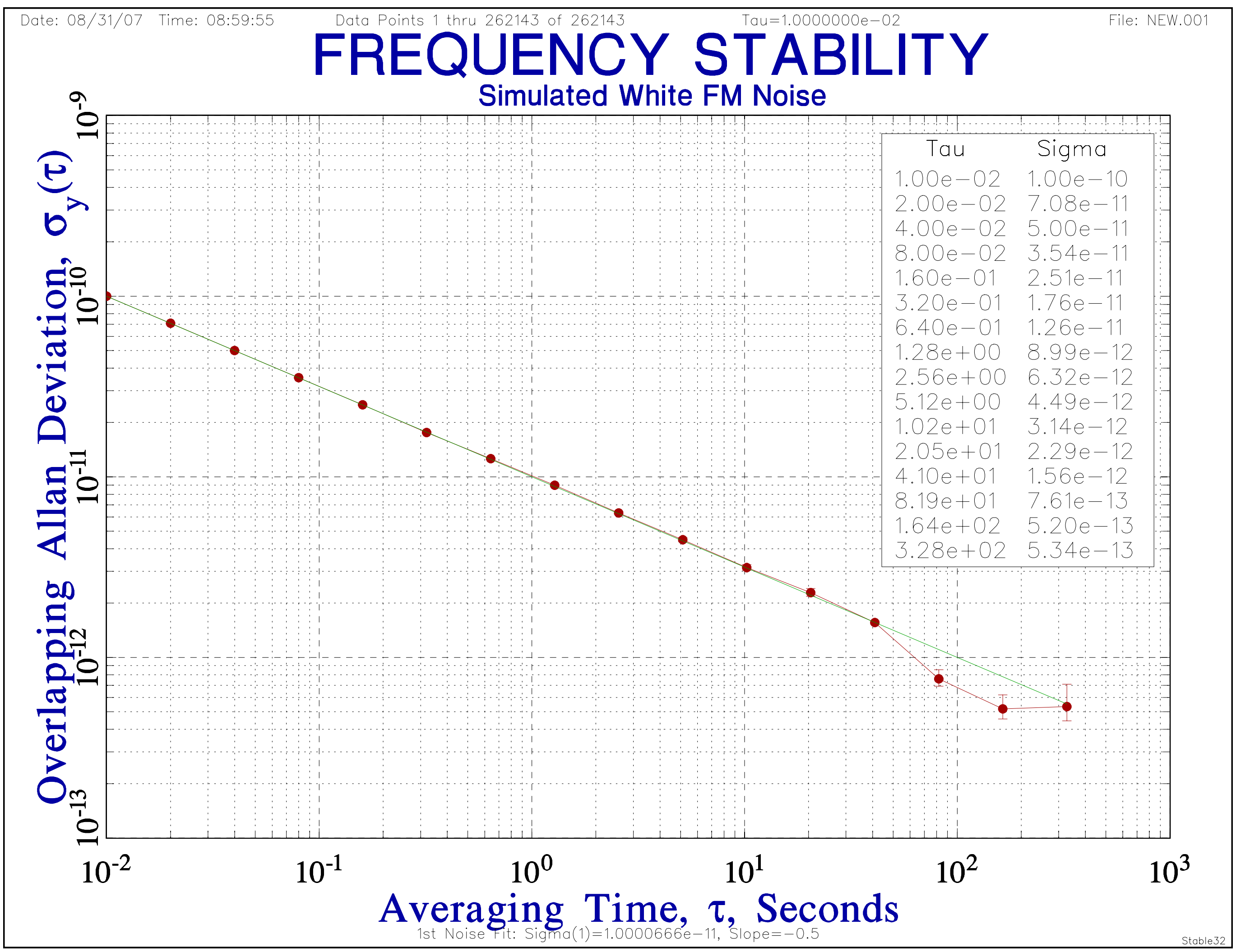


TIME DOMAIN ANALYSIS

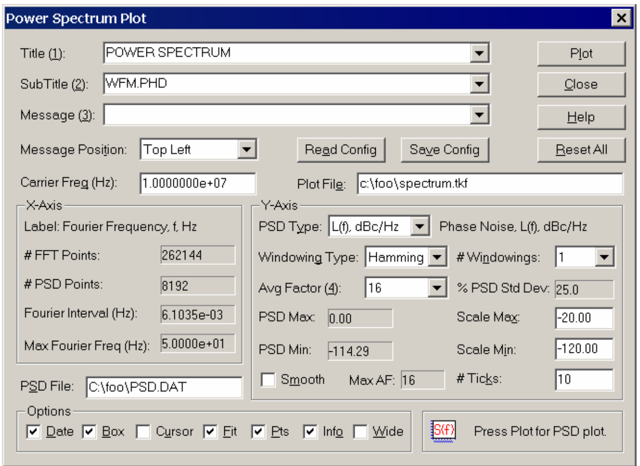


AF	Tau	#	Alpha	Min Sigma	Sigma	Max Sigma
1	1.0000e-02	262142	0	9.9894e-11	1.0005e-10	1.0020e-10
2	2.0000e-02	262140	0	7.0662e-11	7.0794e-11	7.0927e-11
4	4.0000e-02	262136	0	4.9869e-11	4.9992e-11	5.0116e-11
8	8.0000e-02	262128	0	3.5297e-11	3.5415e-11	3.5535e-11
16	1.6000e-01	262112	0	2.4961e-11	2.5076e-11	2.5193e-11
32	3.2000e-01	262080	0	1.7476e-11	1.7589e-11	1.7703e-11
64	6.4000e-01	262016	0	1.2479e-11	1.2591e-11	1.2705e-11
128	1.2800e+00	261888	0	8.8745e-12	8.9864e-12	9.1027e-12
256	2.5600e+00	261632	0	6.2068e-12	6.3173e-12	6.4338e-12
512	5.1200e+00	261120	0	4.3766e-12	4.4864e-12	4.6049e-12
1024	1.0240e+01	260096	0	3.0373e-12	3.1446e-12	3.2642e-12
2048	2.0480e+01	258048	0	2.1783e-12	2.2865e-12	2.4128e-12
4096	4.0960e+01	253952	1	1.5266e-12	1.5594e-12	1.5944e-12
8192	8.1920e+01	245760	0	6.9274e-13	7.6125e-13	8.5517e-13
16384	1.6384e+02	229376	0	4.5597e-13	5.1960e-13	6.2037e-13

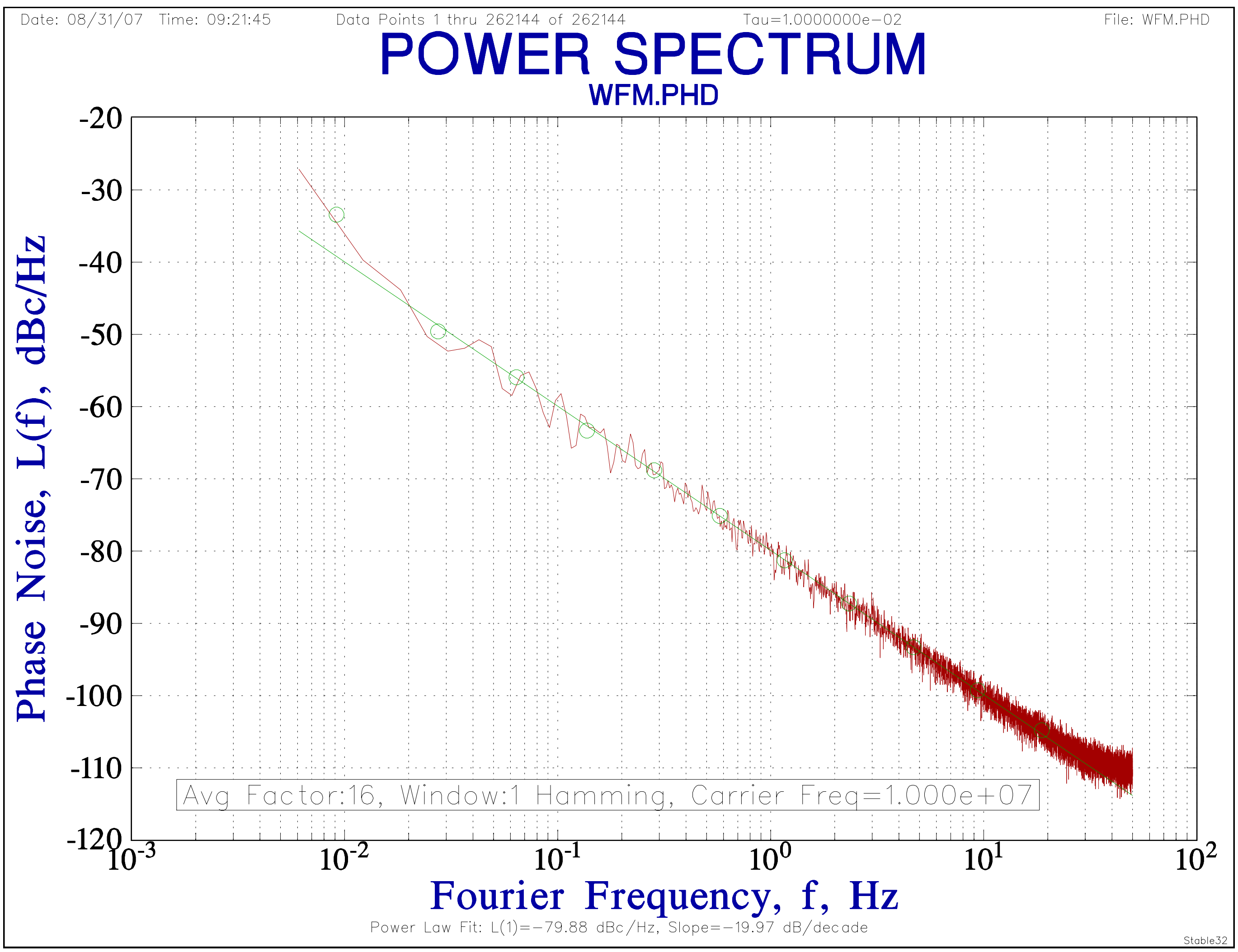
Choose the Run function, select the overlapping Allan variance type and press Calc to begin the analysis. A table of time-domain stability data will be generated. Click on the Lines button, select the Noise line type, and accept the default W FM noise slope by pressing OK. If desired, use the Options button to label the plot with a subtitle "Simulated White FM Noise". Then press the Plot button to produce a stability plot that includes a W FM power-law noise fit. The $\sigma_y(1)$ fit parameter is 1.000e-11, at the desired value.



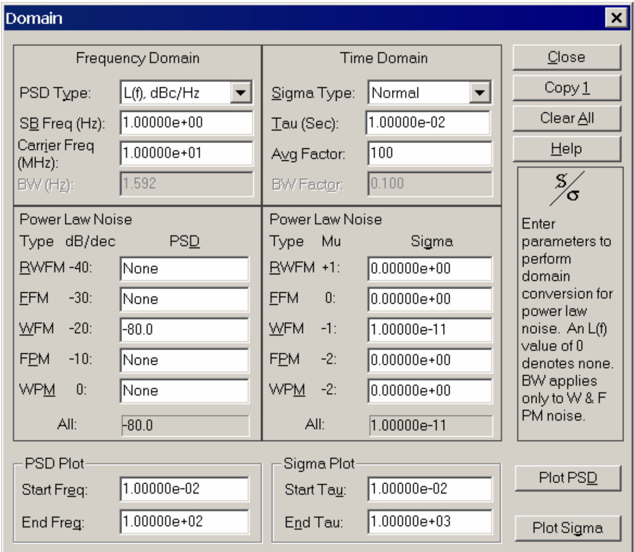
FREQUENCY DOMAIN ANALYSIS



Choose the Power function and calculate a power spectrum for the phase data, using the default 10 MHz carrier frequency and £(f) power spectral density type, the SSB phase noise to signal ratio in a 1 Hz BW as a function of sideband frequency, f. Select one Hamming window and an averaging factor of 16. Press Plot to produce a PSD plot that includes a smoothed fit using octave-band averages. The fit parameters show an £(1) value of –79.9 dBc/Hz and a slope of –20.0 dB/decade, in close agreement with the expected values of –80 dBc/Hz and –20 dB/decade, as shown in the following paragraph.



DOMAIN CONVERSION

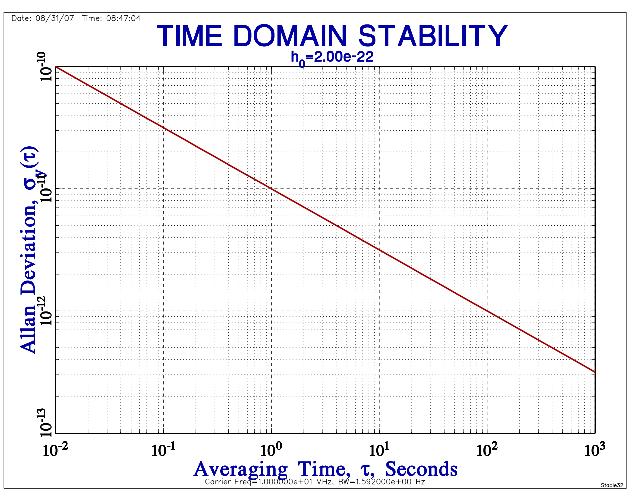


The Stable32 Domain function can be used to determine the PSD values that correspond to the time domain parameters that were used to generate the simulated power-law noise. Press Domain and enter those parameters. Use the normal sigma and enter its 1-second value as 1e-11 with tau =1e-2 and an averaging factor of 100. Choose the £(f) PSD type, a 1 Hz sideband frequency, and use the default 10 MHz carrier frequency. The resulting £(1) value is –80 dBc/Hz, and white FM noise has a slope of –20 dB/decade.

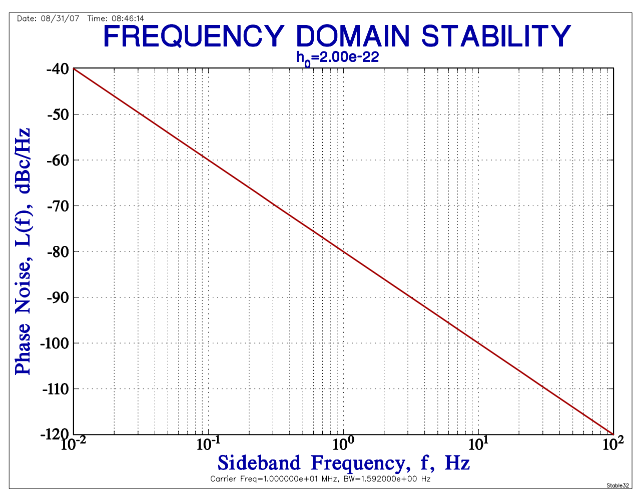
```
Domain Calculation Results:
Frequency Domain                  Time Domain
PSD Type:     L(f), dBc/Hz        Sigma Type:   Normal
SB Freq (Hz):1.00000e+00   Tau (Sec):    1.00000e-02
Carrier(MHz):1.00000e+01   Avg Factor:   100

Power Law Noise
Type   dB/dec        PSD           Type   Mu            Sigma
RWFM   -40:          None          RWFM   +1:           0.00000e+00
FFM    -30:          None          FFM     0:           0.00000e+00
WFM    -20:          -80.0         WFM    -1:           1.00000e-11
FPM    -10:          None          FPM    -2:           0.00000e+00
WPM      0:          None          WPM    -2:           0.00000e+00
All    -80.0         All                                1.00000e-11
```

Time and frequency domain stability plots for this power law noise are shown below:



Time Domain Stability Plot



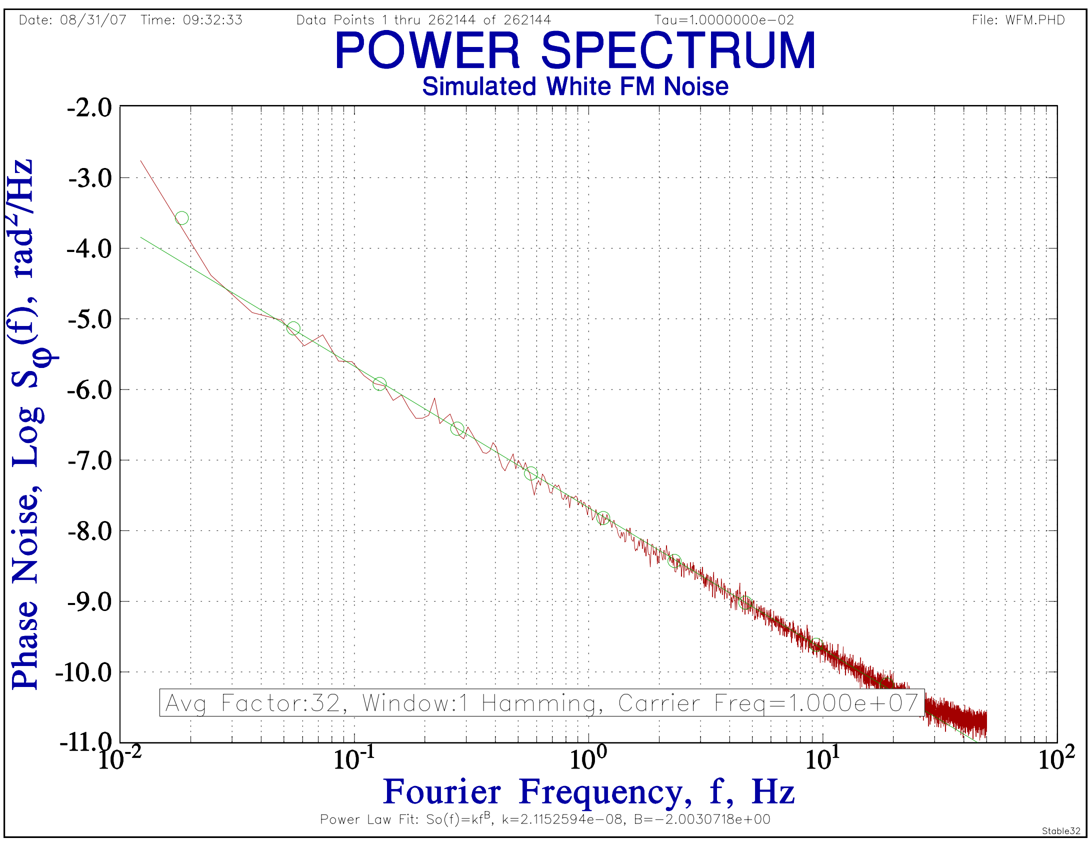
Frequency Domain Stability Plot

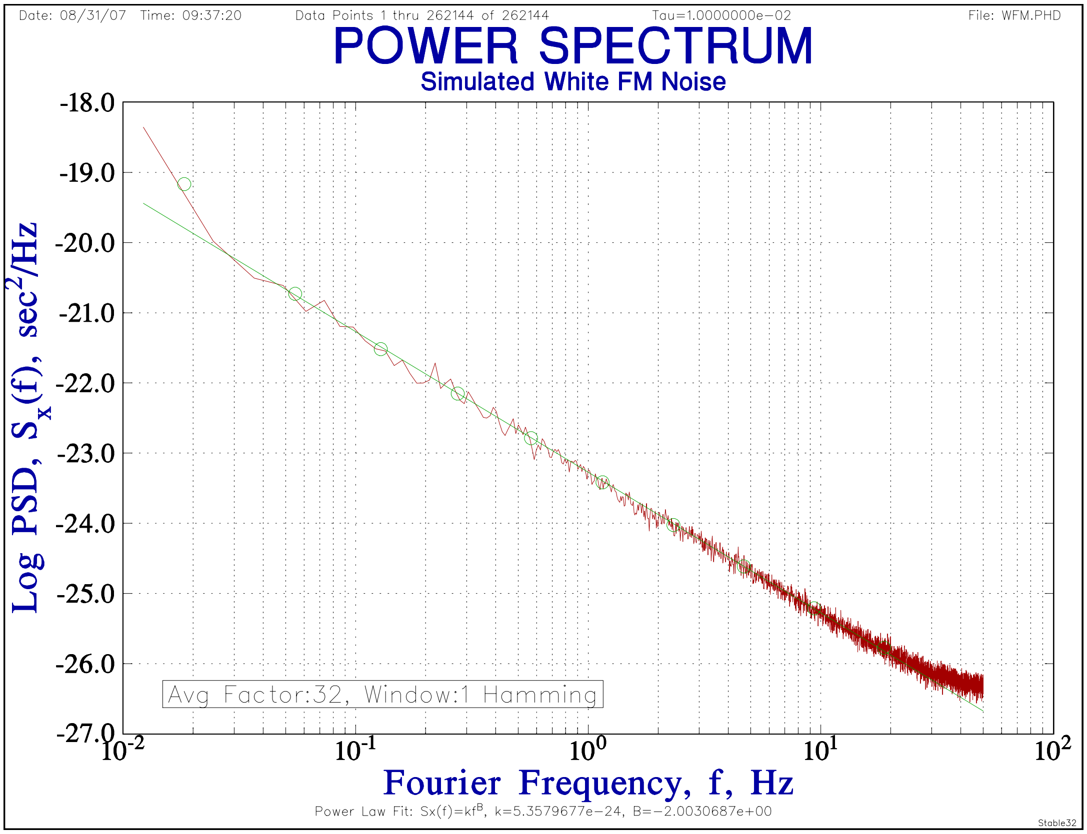
OTHER SPECTRAL TYPES

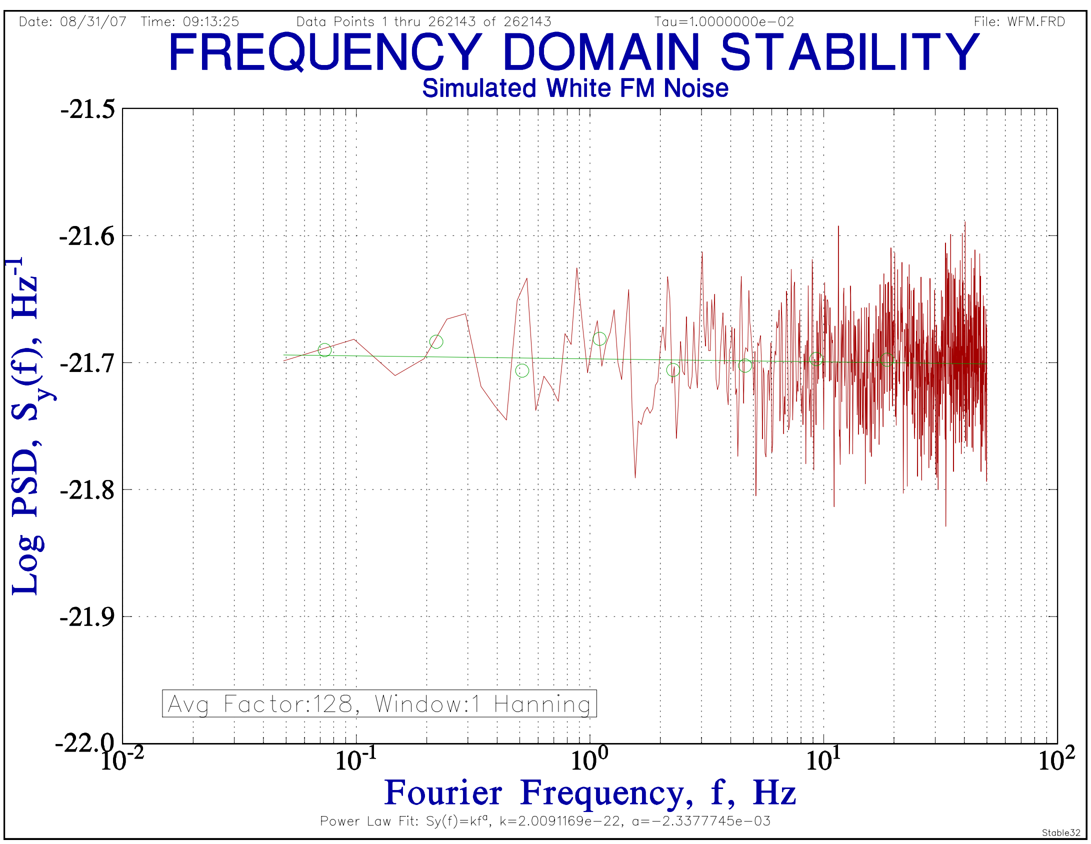
Stable32 supports several other types of PSD that are commonly used for the analysis of frequency domain stability analysis. These are $S_\phi(f)$, the spectral density of the phase fluctuations in rad^2/Hz, $S_x(f)$, the spectral density of the time fluctuations in sec^2/Hz, and $S_y(f)$ the spectral density of the fractional frequency fluctuations in units of 1/Hz.. The Domain function can also be used to determine the expected value of all these quantities for the simulated white FM noise parameters with $\sigma_y(1)$ = 1e-11, τ = 1e-2, and f_o = 10 MHz as shown in the following table.

PSD Type	£(f), dBc/Hz	Sϕ(f), rad^2/Hz	Sx(f), sec^2/Hz	Sy(f), 1/Hz
Data Type	Phase	Phase	Phase	Frequency
Simulated Value	-80	2e-8	5.066e-24	2e-22
Log Value	same	-7.70	-23.3	-21.7
Slope, dB/decade	-20	-20	-20	0
Fit Value	-79.9	2.12e-8	5.36e-24	2.01e-22
Fit Exponent	-2.00	-2.00	-1.96	-0.002

These spectral densities for the simulated data are shown in the plots below:




Date: 08/31/07 Time: 09:37:20
Data Points 1 thru 262144 of 262144
Tau=1.0000000e-02
File: WFM.PHD
POWER SPECTRUM
Simulated White FM Noise
Log PSD, $S_x(f)$, sec^2/Hz
-18.0
-19.0
-20.0
-21.0
-22.0
-23.0
-24.0
-25.0
-26.0
-27.0
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
Avg Factor:32, Window:1 Hamming
Fourier Frequency, f, Hz
Power Law Fit: $Sx(f)=kf^{B}$, k=5.3579677e-24, B=-2.0030687e+00
Stable32


Date: 08/31/07 Time: 09:13:25
Data Points 1 thru 262143 of 262143
Tau=1.0000000e-02
File: WFM.FRD
FREQUENCY DOMAIN STABILITY
Simulated White FM Noise
Log PSD, $S_y(f)$, Hz^{-1}
-21.5
-21.6
-21.7
-21.8
-21.9
-22.0
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
Avg Factor:128, Window:1 Hanning
Fourier Frequency, f, Hz
Power Law Fit: $Sy(f)=kf^{a}$, k=2.0091169e-22, a=-2.3377745e-03
Stable32

PSD OPTIONS

The Stable32 Power function supports a number of options for its power spectral density functions. Bartlett (non-overlapping) periodogram averaging is available with 0-3 Hamming or Hanning windows. The averaging factor may be selected manually or automatically. Multitaper PSDs are available with seven windowing choices. Both the periodogram and multitaper PSDs may be smoothed with a rectangular Fourier frequency window with adjustable parameters. Zero padding may be used as a configuration option. Weighted fits to the PSD results are available. Although intended mainly for the analysis of noise, the Stable32 Power function includes provisions for determining the level of discrete spectral components.

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